JOURNAL OF MATERIALS SCIENCE 13 (1978) 1261–1266

Inelastic behaviour in steel wire pull-out from Portland cement mortar

D. J. PINCHIN*, D. TABOR
Physics and Chemistry of Solids, Cavendish Laboratory, Madingley Road, Cambridge, UK

This paper describes a study of the factors influencing the pull-out force of stainless steel wires embedded in Portland cement mortar. The first part provides a theoretical elastic analysis of the pull-out force in terms of the misfit between wire and matrix, the coefficient of friction at the wire–matrix interface and the elastic constants of the materials. The resulting equation shows that the behaviour is greatly influenced by the Poisson contraction of the wire during pull-out. Since the elastic modulus of the mortar is only five or six times smaller than that of the wire this can lead to a large reduction in stress-transfer across the wire–matrix interface and a corresponding reduction in pull-out force. In general the pull-out force is extremely sensitive to the wire–matrix misfit. The second part deals with an experimental study of the effect of surface finish of the wire and the effect of an externally applied confining pressure. The results show that when a pressure is applied the pull-out force increases, as expected, due to the increase in stress-transfer across the interface. However, a very small amount of movement of the wire leads to a large reduction in pull-out force. This is not due to wear of the matrix; it is due to densification of the cement mortar near the wire surface and is produced by the combined effect of the normal pressure and the tangential traction. This compaction in turn leads to a non-reversible reduction in the fibre–matrix misfit and a fall in pull-out force. The paper concludes with a discussion of the significance of this non-elastic behaviour in the practical performance of fibre reinforced mortars and the role of mechanical deformations of the fibres.

1. Introduction

The mechanical properties of a fibre reinforced cement composite depend strongly on the frictional fibre–matrix stress transfer subsequent to fibre debonding [1]. Numerous studies have been reported in the literature dealing with both the fibre–matrix stress transfer before debonding, the so-called bond strength [2–4], and to a lesser extent with the frictional stress transfer subsequent to debonding [5, 6]. These studies have been of an applied nature and aimed at improving fibre–matrix stress transfer before or after bond failure. Aveston *et al.* [7] and others [1] have shown that composite properties are only slightly affected by improving the fibre bonding and it appears that improved composite performance can only be achieved by improving the frictional stress transfer.

To date little work has been carried out on the mechanism of the frictional fibre–matrix stress transfer and on the fibre–matrix interaction on pull-out. A method has been developed by the authors to increase the fibre–matrix contact pressure and frictional stress transfer during fibre pull-out. This method enables study of the fibre–matrix interaction. The inelastic behaviour observed in these experiments is explained on the basis of a simple theoretical calculation.

---

* Present address: Ontario Research Foundation, Sheridan Park Research Community, Mississauga, Ontario L5K 1B3, Canada.

2. Theory of frictional stress transfer

If we consider a fibre of radius r_f embedded in a cylinder of matrix of radius r_m where the fibre–matrix stress transfer is entirely frictional, we can calculate the stress transferred to the wire by a simple balance of forces. The stress $d\sigma_f$ transferred to the fibre by a frictional shear stress τ_f over a length dx is given by

$$d\sigma_f = \frac{2\pi r_f}{\pi r_f^2} \tau_f dx$$

$$= \frac{2\tau_f}{r_f} dx \,. \qquad (1)$$

If the stress transfer is entirely a frictional process we may write

$$\tau_f = \mu P \qquad (2)$$

where μ is the coefficient of friction and P is the fibre–matrix (normal) contact pressure.

Following Timoshenko [8] the interfacial contact pressure P in a shrink fit configuration with no load in the wire for the condition that $r_m \gg r_f$ is given by

$$P = \frac{\epsilon_o}{(1 + \nu_m)/E_m + (1 - \nu_f)/E_f} \qquad (3)$$

where ϵ_o = the strain in the shrink fit configuration,

ν_m, ν_f = Poisson's ration of matrix and fibre,

and E_m, E_f = Young's modulus of matrix and fibre,

The strain ϵ_o may be due to either shrinkage of the cement or to strain in the cement caused by an externally applied pressure, or to a sum of these: $\epsilon_0 = \epsilon_{\text{shrinkage}} + \epsilon_{\text{pressure}}$. The strain between the wire and the matrix can be expressed as $\epsilon_o = \delta/r_f$ where δ is the wire–matrix misfit, that is the difference between the radius of the wire and the radius of the hole in the matrix in the absence of the wire.

When the fibre is loaded along its length by a stress σ_f it will undergo a Poisson contraction ϵ_f given by

$$\epsilon_f = \frac{\nu_f \sigma_f}{E_f} \,. \qquad (4)$$

This will reduce the interfacial contact pressure caused by the original matrix strain

$$\epsilon = \epsilon_o - \epsilon_f . \qquad (5)$$

Substituting into Equation 1 and solving the resulting differential equation we find the dependence of the stress in the fibre σ_f on the embedded length, x is given by

$$\sigma_f = \frac{\delta E_f}{r_f \nu_f} \times \left[1 - \exp\left(\frac{-2\,\nu_f \mu x}{E_f r_f \{(1 + \nu_m)/E_m + (1 - \nu_f)/E_f\}}\right)\right] \qquad (6)$$

In the case $E_f \gg E_m$ this reduces to the equation derived by Takaku and Arridge [9] for the pull-out of steel wire from epoxy resin where $E_f \gg E_m$.

In the case where the embedded length, x, and coefficient of friction, μ, remain unchanged, the bracketed term in Equation 6 may be treated as a constant, and we may write

$$\sigma_f = K_1 \delta \qquad (7)$$

where

$$K_1 = \frac{E_f}{r_f \nu_f} \times \left[1 - \exp\left(\frac{-2\,\nu_f \mu x}{E_f r_f \{(1 + \nu_m)/E_m + (1 - \nu_f)/E_f\}}\right)\right].$$

The variation in pull-out load (fibre stress × fibre cross-sectional area) can hence be attributed to changes in δ, the fibre–matrix misfit, if μ remains constant.

3. Experimental programme

Ordinary Portland cement mortar with a water/cement ratio of 0.35 and an aggregate/cement ratio of 1.77, cured for 28 days under water, was used for all tests described here. Aggregate used in mortar preparation was Thames Valley flint gravel. Specimens were cylindrical of radius $r_m = 17.25$ mm with a centrally embedded stainless steel 302 wire of 0.87 mm diameter. Preparation of the specimens is fully described elsewhere [10–12]. The wires used in the test were electropolished in an orthophosphoric/sulphuric acid mixture and some were then roughened by blasting with silica beads. Four wire surface conditions were used: (a) centre line average roughness (CLA) = 0.08 μm; (b) CLA = 0.19 μm; (c) CLA = 0.44 μm; (d) CLA = 1.06 μm. Details of these treatments and wire surfaces are also given in Pinchin [10].

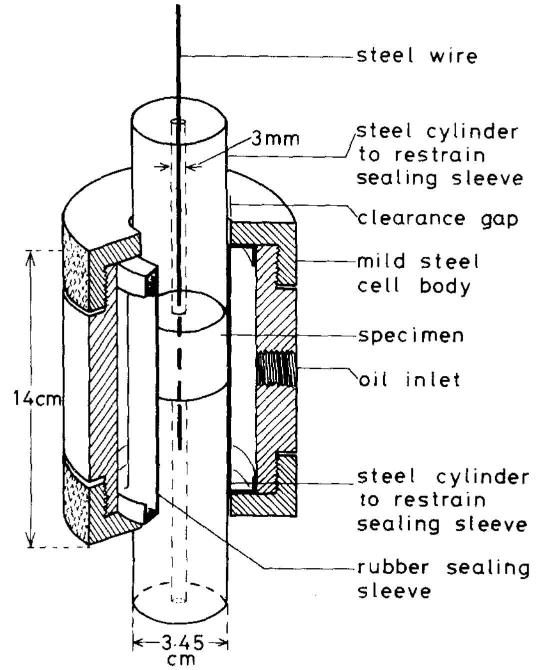

Figure 1 Cut-away view of apparatus for applying confining pressure to cement specimens during pull-out.

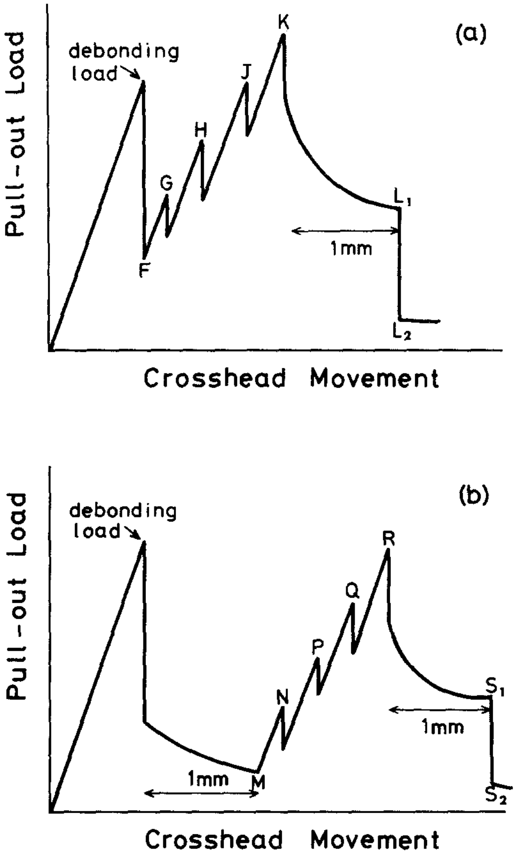

Figure 2 Experimental routine showing effect of confining pressure on wire pull-out identifying positions described in text; (a) pressure applied immediately after debonding, (b) pressure applied after 1 mm wire–matrix slip subsequent to debonding.

The equipment shown in Fig. 1 was used to apply the confining pressure to the specimen. Similar equipment has previously been used to study the bond between cement and glass fibres [12]. The mortar cylinders (3.42 cm diameter, 3.06 cm long) containing the centrally embedded wire were held between two steel end plates and hydraulic pressure could be applied via the rubber sleeve. The wire was loaded with no pressure applied to the specimen until debonding. Pressure was applied either immediately subsequent to debonding (Fig. 2a) or after a cross-head movement of 1 mm (Fig. 2b). The pressure was applied in four stages, 7.5, 14.5, 21.5 and 28.5 $\mathrm{N\,mm^{-2}}$. After each of these pressures was reached the cross-head was moved until wire slip occurred and then immediately stopped for the next increment of pressure and the process repeated. These points are shown as F (zero pressure applied to specimen), G, H, J and K in Fig. 2a, and M (zero pressure applied to specimen), N, P, Q, and R in Fig. 2b. The pressure of 28.5 $\mathrm{N\,mm^{-2}}$ was maintained while the wire was pulled 1.0 mm relative to the matrix (L_1 or S_1). At this stage the pressure was released (L_2 or S_2).

The results of these experiments with wires A, B, C and D are presented in Figs. 3a to d respectively. The pull-out load versus confining pressure (i.e. points F, G, H, J, K or M, N, P, R, S on Fig. 2) is plotted in (i) of each of these diagrams and the pull-out load as a function of wire movement with confining pressure maintained (i.e. K to L_1 or R to S_1) is plotted in (ii) of the corresponding diagram. The results shown are the average results from a minimum of 3 specimens although 4 specimens were tested for most points.

4. Discussion of results

If we consider wire A in Fig. 3a it is apparent that the application of pressure both immediately subsequent to debonding and after wire movement of 1.0 mm results in a marked increase of pull-out load. The coefficient of friction between cement and steel or cement and cement has been found by one of the authors [10] to be independent of both the contact pressure over the range used in these

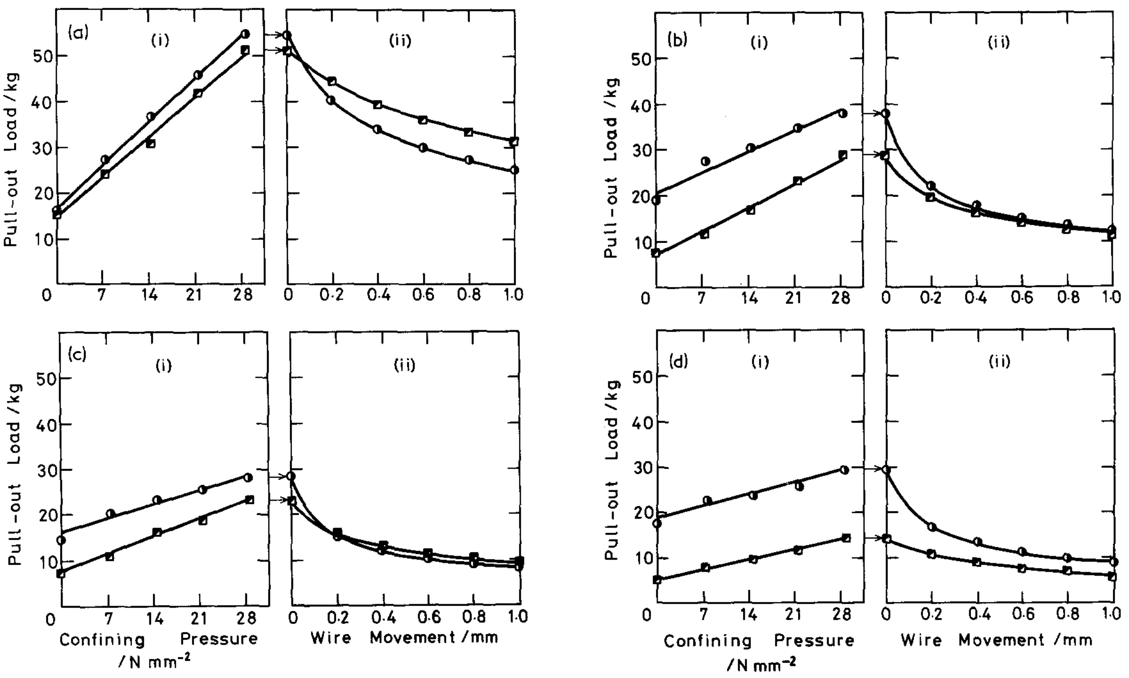

Figure 3 (a), (b), (c) and (d): (i) pull-out load versus confining pressure for wire A, B, C and D respectively; (ii) pull-out load versus wire movement with confining pressure of 28.5 N mm^{-2}. ◑ Pressure applied immediately after debonding. ▨ Pressure applied after 1.0 mm wire–matrix movement subsequent to debonding.

experiments and the amount of fibre–matrix movement. The increase in the pull-out load from approximately 16 kg before application of confining pressure to 52 kg after application of the confining pressure of 28.5 N mm^{-2} must be due solely to the increase in the fibre–matrix misfit. This increase of 36 kg compares with the theoretically predicted increase of 25.4 kg (Equation 6).

Wire movement subsequent to the application of the confining pressure of 28.5 N mm^{-2} causes a marked reduction in the frictional stress transfer and for the reasons given above must also be due to a reduction in the fibre–matrix misfit. Examinations of the wire surface after pull-out shows no detectable removal of material from the wire. The reduction in the fibre–matrix misfit is therefore due to yielding or densification of the cement near the wire surface.

Table I and II give the pull-out load at four stages of the pull-out and the wire matrix misfit (δ in Equation 7) necessary to produce this pull-out load at each of these stages:

(1) before the application of pressure (F or M on Fig. 2);

(2) after pressure of 28.5 N mm^{-2} is applied to the specimen (K or R on Fig. 2),

(3) after the wire has moved 1.0 mm under a pressure of 28.5 N mm^{-2} (L_1 or S_1 on Fig. 2),

(4) after the pressure is released (L_2 or S_2 on Fig. 2).

Table I presents the results when the confining pressure is applied immediately subsequent to debonding and Table II gives the same results after a wire–matrix movement of 1.0 mm prior to application of the confining pressure.

From Fig. 3a and Tables I and II it is clear that the frictional fibre–matrix stress transfer is similar at each stage of pull-out both immediately subsequent to debonding and after movement of 1.0 mm. The increase of the fibre–matrix stress transfer on wire movement while under pressure and on release of pressure are similar for both cases.

The increased roughness of wires B, C and D causes a marked change in the behaviour described above. The frictional stress transfer before the application of the confining pressure decreases markedly in the first mm of wire movement for wires B, C and D so that there is a considerable difference in frictional stress transfer at points F and M on Fig. 2. The increase in pull-out load with confining pressure is also much less pronounced

TABLE I Pull-out load and wire–matrix radius misfit to produce corresponding pull-out load for wires A, B, C and D at various stages of pull-out. Pressure applied immediately subsequent to debonding.

Stage of pull-out	Wire A		Wire B		Wire C		Wire D	
	Pull-out Load (kg)	Wire–matrix radius misfit (μm)	Pull-out load (kg)	Wire–matrix radius misfit (μm)	Pull-out load (kg)	Wire–matrix radius misfit (μm)	Pull-out load (kg)	Wire–matrix radius misfit (μm)
Before application to pressure	16.4	0.17	19.3	0.20	14.5	0.15	17.6	0.19
After application of pressure of 28.5 N mm^{-2}	54.6	0.58	37.6	0.40	28.1	0.30	29.7	0.31
After wire Movement of 1.0 mm under pressure	25.0	0.26	12.0	0.13	8.4	0.09	9.1	0.10
After release of pressure	2.7	0.03	1.8	0.02	2.0	0.03	2.4	0.03

CLA values: A, 0.08 μm; B, 0.19 μm; C, 0.44 μm; D, 1.06 μm.

than with wire A. Taking wire D as an example the increases in pull-out load with application of 28.5 N mm^{-2} confining pressure is only 12.1 kg immediately subsequent to debonding and 9.6 kg after movement of 1 mm. These correspond to an increase in the wire–matrix misfit of 0.12 and 0.11 μm compared with the theoretically predicted 0.27 μm. The interaction of the asperites on the roughened wires with the cement matrix must cause a more rapid rate of compaction or densification of the cement near the interface, compared with wire A. The mechanical compaction of hydrated cement paste on a large scale has been used by Sereda and co-workers [14, 15] to reduce the porosity and increase the strength of set cement paste. The compaction discussed in this paper occurs on a much finer scale near the embedded wire and is associated with the work done by the

TABLE II Pull-out load and wire–matrix misfit to produce corresponding pull-out load for wire A, B, C and D at various stages of pull-out. Pressure applied after 1 mm wire–matrix movement.

Stage of pull-out	Wire A		Wire B		Wire C		Wire D	
	Pull-out load (kg)	Wire–matrix radius misfit (μm)	Pull-out load (kg)	Wire–matrix radius misfit (μm)	Pull-out load (kg)	Wire–matrix radius misfit (μm)	Pull-out load (kg)	Wire–matrix radius misfit (μm)
Before application of pressure	15.9	0.17	7.5	0.08	7.4	0.08	5.2	0.05
After application of pressure of 28.5 N mm^{-2}	51.2	0.54	28.7	0.30	23.1	0.24	14.8	0.16
After wire movement of 1.0 mm under pressure	31.6	0.33	11.9	0.13	9.8	0.10	5.4	0.06
After release of pressure	3.6	0.04	2.3	0.02	2.4	0.03	1.3	0.01

CLA values: A, 0.08 μm; B, 0.19 μm; C, 0.44 μm; D, 1.06 μm.

wire as it slides over the matrix: this is accentuated by the presence of surface asperites on the wire. The compaction is of the order of 0.1 to 0.3 μm and is therefore too small to detect by any of the means available to the authors.

Compaction of cement on sliding over a steel surface has been measured in a standard frictional experiment by one of the authors [10] and again shows that the compaction is due to the sliding process and not to the normal load. The geometry of the test and the contact pressures were very different from those obtained in the pull-out tests described here and indeed showed a much higher degree of compaction.

5. Significance of results

The compaction of the cement matrix will have some significant effects both on theories of fibre reinforced cement and on actual composite properties. On the theoretical side, elastic analyses concerning frictional fibre–matrix stress transfer and the effect of a wire taper (for example that due to Hale [16]) will require modification to allow for this non-elastic behaviour. On the practical side, in the absence of some mechanical anchorage the frictional fibre–matrix stress transfer will decrease on wire–matrix movement; that is in the course of pull-out during composite cracking and during crack widening. This will result in a reduced post-cracking strength and a decrease in composite energy absorption. The advantages of a mechanical anchorage of the fibre becomes obvious when this is considered. The magnitude of mechanical deformation of the wire required to produce a large increase in the fibre–matrix stress transfer is in the authors' opinion usually more than is necessary. For example, it is seen from Equation 3, that if the wire radius at some point along its length is increased by 1% by mechanical deformation, the interfacial pressure is increased by approximately 250 N mm^{-2}. This will not only cause local matrix yielding but will also provide a large fibre–matrix stress transfer even allowing for matrix compaction. This small deformation may be compared with the large deformation (20% to 30% change in radius) generally found in commercially available shaped fibres.

Acknowledgements

The work described has been carried out under a contract sponsored by the Building Research Establishment, and is published by permission of the Director. The authors would like to thank Dr E. A. Yoffe, Dr A. J. Majumdar and Miss V. Laws for many helpful discussions during the course of the work.

References

1. V. LAWS, P. LAWRENCE and R. W. NURSE. *J. Phys. D: Appl. Phys.* 6 (1973) 523.
2. R. C. DE VEKEY and A. J. MAJUMDAR, *Magazine of Concrete Research* **20** (1968) 229.
3. L. J. DEN BOER, Procedings of the Conference on Properties and Applications of Fibre Reinforced Concrete and other Fibre Reinforced Building Materials, Delft (1973) p. 97.
4. G. H. TATTERSAL and C. R. URBANOWICZ, *Magazine of Concrete Research* **26** (1974) 105.
5. A. E. NAAMAN and S. P. SHAH, RILEM Symposium, London (1975) p. 171.
6. M. MAAGE, Ph.D. Dissertation, University of Trondheim (1976).
7. J. AVESTON, R. A. MERCER and J. M. SILLWOOD, National Physical Laboratory Report, SI No. 90/11/98 (1975).
8. S. TIMOSHENKO. "Strength of Materials" (MacMillan, London, 1941).
9. A. TAKAKU and R. G. C. ARRIDGE, *J. Phys. D: Appl. Phys.* 6 (1973) 2038.
10. D. J. PINCHIN, Ph.D. Dissertation University of Cambridge (1977).
11. D. J. PINCHIN and D. TABOR, *Cement and Concrete Research* 8 (1978) 15.
12. *Idem, ibid.* 8 (1978) 139.
13. A. J. MAJUMDAR and V. LAWS, Private Communication, Building Research Establishment (1974).
14. P. J. SEREDA, R. F. FELDMAND and E. G. SWENSON, Symposium on Structure of Portland Cement Paste and Concrete, Highway Research Board, Special Report 90 (1966) p. 58.
15. I. SOROKA and P. J. SEREDA, Proceedings of the 5th International Symposium on the Chemistry of Cement, Tokyo **3** (1968) p. 67.
16. D. K. HALE, RILEM Symposium London (1975) p. 159.

Received 31 August and accepted 19 October 1977.